Synopsis for PhD Programme

Title of the Research:

Some model studies on solute dispersion through arterial vessels

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Dispersion represents transport of solute by convective velocity of the fluid and molecular diffusion. This solute can be drug, LDL, HDL, calcium etc. When a mass injected into blood stream, it flows along blood, then it is important to determine the concentration of mass in the lumen as well as in the wall tissue, because at some point it may be toxic above a threshold level. The analysis of solute dispersion in Newtonian or non-Newtonian fluid flow through a tube with wall reaction has wide application in the field of biomedical engineering, chemical engineering, physiological fluid dynamics, and environmental science.

In 1953, Taylor [1] first described the method of solute dispersion in a steady laminar viscous fluid flow through a circular tube analytically. He calculated the effective molecular diffusion coefficient (D_{eff}) as $D_{\text{eff}} = R^2 w_m^2/48D_m$, where D_m is the molecular diffusivity, w_m is the mean axial velocity and R is the radius of the tube. Aris in 1956 [2] modified the diffusion equation by adding the molecular diffusivity term D_m and the effective molecular diffusion coefficient becomes $D_{\text{eff}} = D_m + R^2 w_m^2/48D_m$. The solution of the complete convective-diffusion equation was obtained numerically by Ananthakrishnan et al. [3] and according their analysis, Taylor-Aris's theory was applicable only when the time after injection of the solute exceeded about $.5(R^2/D_m)$.

Aris, by generalising Taylor's approach introduced a new technique named, "Method of moments", to determine diffusion coefficient. Later, Barton [4] again modified the method of moment. The Aris-Barton approach is very effective to determine the dispersion coefficient, long time after input of solute. In 1970, Gill and Sankarasubramanian [5] described the exact analysis of unsteady convective diffusion. In this study, fully developed steady laminar flow in a tube was considered and analysed the dispersion of injected solute at each time.

Further, in 1973, Sankarasubramanian and Gill [6] derived a generalised dispersion model for unsteady convective diffusion with mass transfer in the boundary wall. In this model, three effective transport coefficient was considered i.e. exchange coefficient, convective coefficient and dispersion coefficient to determine the concentration of solute. Sankarasubramanian and Gill model is applicable for short as well as long time. The exchange coefficient arises due to the irreversible reaction at the boundary, the convection coefficient occurred due to the velocity of the solute and the dispersion coefficient occurred due to the molecular diffusion and velocity of the fluid. Transport coefficients were functions of time and the dispersion coefficient depends on time, initial solute dispersion and the chemical reaction of the solute, as showed by DeGance and Johns [7].

Study of shear augmented dispersion of a solute in a Casson fluid flow in a conduit was carried out by Dash et al [8] using the model of solute dispersion of Gill and Sankarasubramanian. They showed that effective diffusivity depends on time. Effective diffusivity increase with time, but upto a critical time $(.48R^2/D_m)$ for dispersion in a pipe. Using the method of moment, Mazumder and Das [9] analysed the axial dispersion of a passive contaminant in a pulsatile fluid flow in a tube with wall reaction.

Many researcher used generalised dispersion model proposed by Sankarasubramanian and Gill, to analyse the nature of solute dispersion in many situations with or without boundary reaction. Hazra et al. [10] have considered oscillatory non-Newtonian fluid flow and obtained longitudinal dispersion coefficient using generalised dispersion model with the effect of frequency of pressure and viscoelastic parameter. The effect of irreversible tube wall reaction in the dispersion of a tracer for an annular flow was described by Sankar and Jayaraman [11] using generalised dispersion model. Later, by using the similar model, Nagarani et al. [12] described the solute dispersion in a non-Newtonian Casson fluid flow in a tube and channel with the combined effect of irreversible wall reaction and yield stress.

Multi-scale asymptotic analysis was done by Ng [13] to determine the solute dispersion. In this analysis both reversible and irreversible wall reaction have considered in both steady and oscillatory flow. Using Aris-Barton method, Kumar and Jayaraman (2008) [14] described the dispersion process by considering oscillatory flow in curved channel with wall absorption. In 2009, Nagarani et al. [15] extended their previous work by applying Casson fluid model to catheterized artery with boundary absorption. But their approach is applicable only for large time after input of mass in fluid. Effect of yield stress, Womersley frequency number, Schmidt number and pressure component on solute dispersion in Casson fluid flow through a tube was described by Nagarani and Sebastian [16]. In pulsatile Casson fluid flow through a tube, dispersion of solute at each time level was analysed by Rana and Murthy [17].

In most of the research works cited above, the change in solute concentration in the lumen due to boundary reactions have been discussed only. Very few literatures are available that discuss the change in solute concentration in the lumen as well as in the wall tissue. Recently distribution and retention of drug through an atherosclerotic plague from a half-embedded stent was discussed by Mandal and Mandal [18]. Arterial drug transport from a half-embedded stent was analysed by Mandal et al. [19].

Motivated from above literature, we are interested to study the solute dispersion process in the lumen as well as in the tissue of the artery. In this study, solute is entered into the lumen through luminal inlet- relevant to intra-vascular drug delivery. This study might help to analysed the drug concentration in the fluid phase and in the tissue phase where the lesion or atherosclerosis is occurred. The fluid can be Newtonian or non-Newtonian with laminar or oscillatory flow through an artery with various boundary reaction. We are also interested to study the effect of interstitial fluid flow on the dispersion of solute in the tissue. Transient transport of the drug and momentum in presence of flow pulse may be studied to examine its roll on the effectiveness of delivery. Retention of drug with the artery tissue may be examined for varied physiological condition. In order to validate our findings, we aim to compare our results with existing ones available in the literature.

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